THEORY OF REGENERATIVE FREQUENCY DIVIDERS USING DOUBLE-BALANCED MIXERS

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ABSTRACT

Regenerative frequency halvers using double-balanced mixers are analysed in terms of modified Bessel functions. Closed-form solutions predict the threshold of turn on, the steady-state input-output amplitude relationship, and the operational bandwidth.

INTRODUCTION

There has been a recent resurgence of interest in regenerative frequency dividers, using both silicon bipolar [1] and gallium arsenide MESFET [2-4] technologies. Although their history is long [5-7], a realistic theory has been lacking. This paper analyses a regenerative divider using a zero-memory double-balanced Schottky diode mixer (DBM), see Fig. 1.

For regeneration, a finite signal, e.g. due to thermal noise, must be present in the loop, and the loop gain must be greater than unity. Additionally, for zero output with no input the loop gain must be less than unity when the input signal is removed [3]. The input signal at 2ω is mixed with a feedback signal at ω to give sidebands at ω and 3ω. The bandpass filter (BPF) selects the lower sideband. The amplifier compensates for loop losses.

For stable microwave performance with FM inputs, the loop delay must be much less than the FM modulation period. For CW inputs, the loop delay limits the band of stable operation, which is bounded by submultiple division modes [8], and regions of chaotic behaviour.

Analysis

In Fig. 1, the input signal is

$$v_{in}(t) = V_{in}\cos(2\omega t) \tag{1}$$

while the output signal at ω is assumed to be

$$v_{out}(t) = V_{out}\cos(\omega t+\phi_1) \tag{2}$$

where ϕ_1 is the BPF phase shift. If the RF input voltage V_{RF} to a diode double-balanced mixer is small compared with the LO input voltage V_{LO}, then the IF output current of the DBM can be written [9]

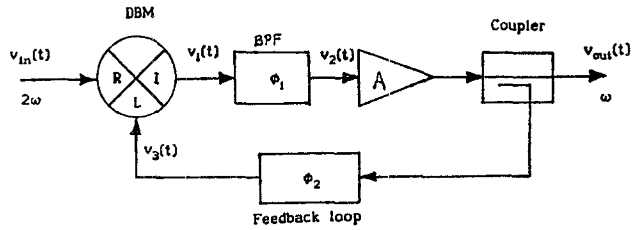

Fig. 1 Schematic of a feedback frequency halver using a double-balanced mixer (DBM).

$$i_{IF}(t) = 4I_sX_R\sum_{N=1}^{\infty}\{I_{2N-1}(X_L).(\cos[(2N-1)\omega_L+\omega_R]t + \cos[(2N-1)\omega_L-\omega_R]t)\} \tag{3}$$

where

$$\left.\begin{aligned} I_s &= \text{diode saturation current},\\ I_K(.) &= \text{modified Bessel function of order } K,\\ X_R &= \frac{qV_{RF}}{nKT},\\ X_L &= \frac{qV_{LO}}{nKT},\\ V_{RF}, V_{LO} &= \text{amplitudes of RF and LO voltages},\\ \omega_R, \omega_L &= \text{RF and LO angular frequencies}. \end{aligned}\right\} \tag{4}$$

The input signal at 2ω is applied to the RF port of the DBM, the feedback signal v_3 at frequency ω to the LO port;

$$v_3(t) = V_3\cos(\omega t+\theta) \tag{5}$$

so that

$$V_R = V_{in},\quad V_L = V_3,\quad \omega_R = 2\omega,\quad \omega_L = \omega.$$

Assuming that:

(i) the DBM IF port is terminated in R_L,

(ii) the filter has a pass-band gain of G, passes signals at ω only, and has a phase shift $\phi_1 = \phi_{o1} + (\omega-\omega_o)t_{o1}$.

(iii) the linear amplifier and coupler have an overall gain of A and negligible delay,

(iv) the lossless feedback path has a phase-shift

$$\phi_2 = \phi_{o2} + (\omega-\omega_o)t_{o2} .$$

then

$$v_3(t) =$$
$$4AGR_L I_s X \left[\mathbb{I}_1(Y)\cos(\omega t-\theta+\phi) + \mathbb{I}_3(Y)\cos(\omega t+3\theta+\phi) \right] , \quad (7)$$

Equating (6) and (7) yields the amplitude solution:

$$\boxed{X(\omega) = \frac{Y}{k} \cdot \frac{\sqrt{[\mathbb{I}_1(Y) + \mathbb{I}_3(Y)]^2 - 4\mathbb{I}_1(Y)\mathbb{I}_3(Y)\cos^2\phi(\omega)}}{\mathbb{I}_1^2(Y) - \mathbb{I}_3^2(Y)}} \quad (8)$$

where $k = 4AGR_L I_s q/(kT)$, and the phase solution:

$$\boxed{\theta = \frac{1}{2}\arctan\left[\frac{\mathbb{I}_1(Y) + \mathbb{I}_3(Y)}{\mathbb{I}_1(Y) - \mathbb{I}_3(Y)} \cdot \tan\phi \right]} \quad (9)$$

where

$$\phi(\omega) = \phi_{o1}+\phi_{o2} + (\omega-\omega_o)(t_{o1}+t_{o2}) \quad (10)$$

is the frequency-dependent total loop phase angle.

Starting condition

Regeneration normally starts from thermal noise level, so the threshold drive level is found from (8) as $Y \rightarrow 0$:

$$X_{threshold} =$$
$$\lim_{Y\to 0}(X) = \lim_{Y\to 0}\left[\frac{Y}{k} \cdot \frac{1}{\mathbb{I}_1(Y)} \right] = \frac{2}{k} \quad (11)$$

which is independent of ω except via variations in k, e.g. due to amplifier frequency response. The corresponding phase condition is

$$\theta_{threshold} = \frac{1}{2}\phi(\omega) . \quad (12)$$

Bandwidth

Since the stability is threshold-dependent, a small-signal analysis such as [10] is valid. This shows that the solution is stable only if the total loop phase angle is constrained by

$$\left(-\frac{\pi}{2} + 2\pi K\right) \le \phi(\omega) \le \left(\frac{\pi}{2} + 2\pi K\right), \quad K = \text{integer} \quad (13)$$

which leads to the following bandwidth expression:

$$\omega_o - \frac{\phi_o}{t_o} + \frac{\pi}{t_o}\left[2K - \frac{1}{2}\right] \le \omega \le \omega_o - \frac{\phi_o}{t_o} + \frac{\pi}{t_o}\left[2K + \frac{1}{2}\right] . \quad (14)$$

This means that the available band is periodically subdivided into stable and unstable regions, each of width $1/t_o$, where t_o is the total loop group delay. The number of such regions depends on the bandwidth of the BPF.

Theoretical Response

Equations (8) and (13) yield the theoretical amplitude-frequency response surface of Fig. 2. The input turn-on threshold given by (11) is indicated. As shown in Fig. 3, the surface repeats at loop phase-angle intervals of 2π.

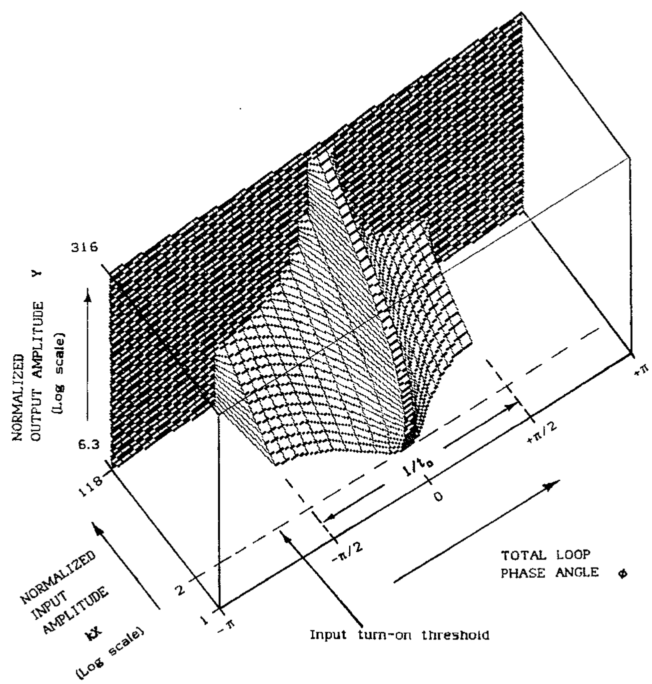

Fig. 2 Theoretical response surface for the regenerative frequency halver using a double-balanced mixer. The total loop group delay is t_o. Note the logarithmic scales.

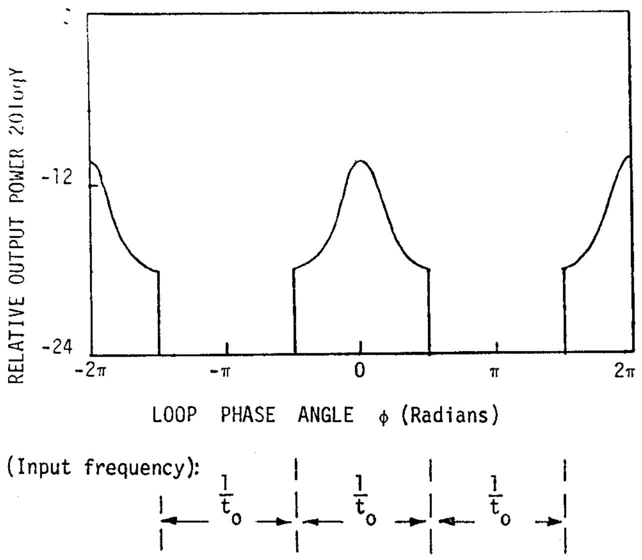

Fig. 3 Repetitive theoretical response of the regenerative frequency halver when the loop bandwidth is >> $1/t_0$.

EXPERIMENTAL RESULTS

Figure 4 shows regions of stable division for a discrete-component regenerative halver using a wideband loop. Apart from the anomalous region around 8 GHz, the periodicity indicates a total loop group delay of 7.3 ns. The small variations of threshold level with frequency are due to variations of total loop gain. Figure 5 shows output power versus input frequency. Apart from amplitude variations, again due to gain fluctuations, the response confirms the predictions of Fig.3.

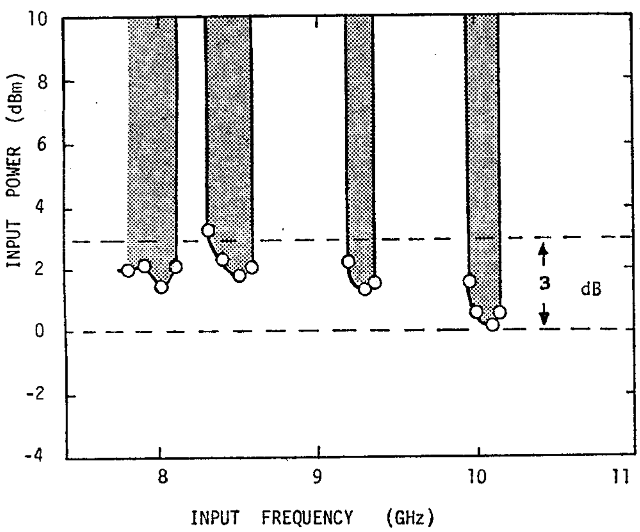

Fig. 4 Measured regions of stable frequency division for a wideband discrete-component regenerative halver.

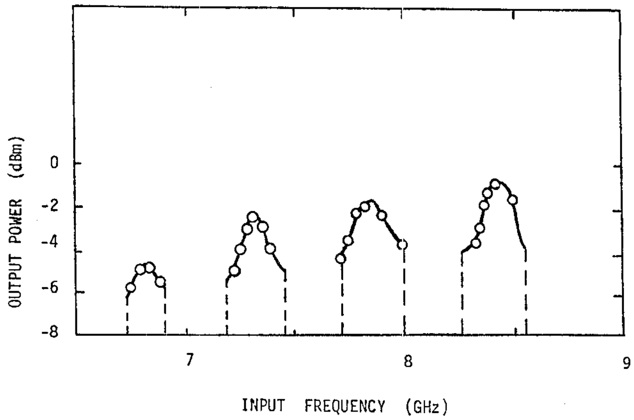

Fig. 5 Measured output power versus input frequency for a feedback halver with long loop delay.

Figure 6 depicts the measured transfer function of the halver at the centre of a stable band; the threshold behaviour is similar to Fig.2, while the limiting action is attributed to mixer diode series resistance and amplifier saturation effects not included in the analysis.

SUMMARY AND CONCLUSIONS

A simple algebraic analysis has for the first time given closed-form expressions which account for all the important features of regenerative frequency halvers that use double-balanced Schottky-diode mixers to obtain input-output isolation.

The method is easily modified to account for other loop configurations.

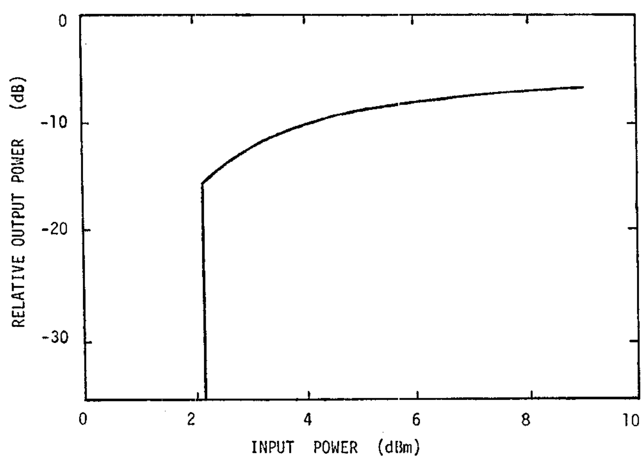

Fig. 6 Measured transfer function of a regenerative halver, showing the threshold of turn-on at the band-centre ω_0 .

ACKNOWLEDGEMENTS

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